Hidden mass in the asteroid belt

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Submitted to *Icarus*

Submitted 13 November 2001

Revised 11 January 2002

Published: Icarus 158, 98–105 (2002)

Manuscript Pages: 33

Tables: 4

Figures: 4

Proposed Running Head: HIDDEN MASS IN THE ASTEROID BELT

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ABSTRACT

The total mass of the asteroid belt is estimated from an analysis of the motions of the major planets by processing high precision measurements of ranging to the landers Viking-1, Viking-2, and Pathfinder (1976–1997). Modeling of the perturbing accelerations of the major planets accounts for individual contributions of 300 minor planets; the total contribution of all remaining small asteroids is modeled as an acceleration caused by a solid ring in the ecliptic plane. Mass M_{ring} of the ring as well as its radius R are considered as solve-for parameters. Masses of the 300 perturbing asteroids have been derived from their published radii based mainly on measured fluxes of radiation, making use of the corresponding densities. This set of asteroids is grouped into three classes in accordance with physical properties and then corrections to the mean density for each class have been estimated in the process of treating the observations. In this way an improved system of masses of the perturbing asteroids has been derived.

The estimate $M_{ring} \approx (5 \pm 1) \cdot 10^{-10} M_{\odot}$ is obtained ($M_{\odot}$ is the solar mass) whose value is about one mass of Ceres. For the mean radius of the ring we have $R \approx 2.80$ AU with 3% uncertainty. Then the total mass M_{belt} of the main asteroid belt (including the 300 asteroids mentioned above) may be derived: $M_{belt} \approx (18 \pm 2) \cdot 10^{-10} M_{\odot}$. The value M_{belt} includes masses of the asteroids which are already discovered, and the total mass of a large number of small asteroids — most of which cannot be observed from the Earth. The second component M_{ring} is the hidden mass in the asteroid belt as evaluated from its dynamical impact onto the motion of the major planets.

Two parameters of a theoretical distribution of the number of asteroids over their masses are evaluated by fitting to the improved set of masses of the 300 asteroids (assuming that there is no observational selection effect in this set). This distribution is extrapolated to the whole interval of asteroid masses and as a result the independent estimate $M_{belt} \approx 18 \cdot 10^{-10} M_{\odot}$ is obtained which is in excellent agreement with the dynamical finding given above.

These results make it possible to predict the total number of minor planets in any unit interval of absolute magnitude H. Such predictions are compared with the observed distribution; the comparison shows that at present only about 10% of the asteroids with the absolute magnitude $H < 14$ have been discovered (according to the derived distribution, about 130000 such asteroids are expected to exist).

Key Words: Asteroids, mass; Planetary dynamics; Radar

I. INTRODUCTION

At present quite accurate measurements of ranging to the martian landers Viking-1,2, and Pathfinder (with the typical error of about 7 m) are available. On this level of accuracy the ephemeris of Mars is very sensitive to perturbations from many minor planets. In the advanced ephemerides of major planets DE403 (Standish *et al.* 1995) and DE405 (Standish 1998) the perturbations from 300 asteroids have been taken into account that made it possible to process these measurements successfully. Nevertheless the total perturbations from other asteroids do affect the ephemerides on a measurable level. A direct computation of perturbations from all asteroids which have been discovered up to the present epoch (in number about 130000) cannot be reliably carried out due to the uncertainties of the individual masses. Moreover it is natural to suppose that the total mass of the asteroids which have not yet been discovered is also large enough to affect the ephemerides of the major planets (though the individual perturbations caused by any one of them are negligible). Mostly these objects are too small to be observed from the Earth but it appears that the total hidden mass may be detected if one studies their impact on the motion of Mars.

A prevailing part of these celestial bodies moves in the asteroid belt and their instantaneous positions may be considered homogeneously distributed along the belt. Thus it seems reasonable to model the perturbations from the remaining small asteroids (for which individual perturbations are not accounted for) by computing additional perturbing accelerations as being caused by a massive ring with a constant mass distribution in the ecliptic plane. Two parameters that characterize the ring (its mass and radius) must be included in the set of solve-for parameters while processing the lander ranging data are processed. In this paper results of such processing are described in detail.

We have used the lunar-planetary integrator embedded in the program package ERA (Krasinsky and Vasilyev 1997). The integrator makes it possible to integrate simultaneously barycentric equations of motion of the nine major planets, Sun, and Moon, equations of the lunar physical libration, and reduced equations of 300–350 minor planets. The mutual perturbations for the five largest asteroids are accounted for; those for the other asteroids are neglected. In the present study the model has been improved by adding the perturbations from the homogeneous ecliptic ring. At the first step its mass has been set equal to zero, to be evaluated by fitting to the observations.

For our analysis it is quite important to use the most accurate available values of the masses of the largest 300 perturbing asteroids. In the next section the method used to derive these masses is outlined.

II. MASSES OF 300 LARGEST ASTEROIDS

There are two groups of methods that allow us to evaluate the masses of asteroids. The first group is hereafter referred to as the astrophysical group. These methods are based on measurements of the flux of radiation from the asteroid and on spectral observations which give its spectral class. The important factor that affects the flux is the radius of the asteroid; other factors may be modeled with sufficient accuracy (Morrison and Lebofsky 1979). Having obtained the spectral class one can attribute a taxonomic type to the asteroid; after that a corresponding density of the asteroid may be related to this type. With the known radius and density, the mass of the asteroid may be easily derived.

Important additional information on asteroid radii is provided by observations of occultations of stars by minor planets (Millis and Dunham 1989), and by radar observations of minor planets of the main belt (which has become a routine procedure — at present 30 asteroids of the main belt are measured in this way; see Magri *et al.* 1999). Comparison with the radar results has confirmed the infrared astronomical satellite (IRAS) radii at the 10% level excepting asteroid 393 Lampetia for which the error is about 30%.

In the methods of the second group the mass of the asteroid has to be estimated from its perturbations upon the motion of some other celestial body. These methods can be applied in the following cases:

1. The perturbed body is another asteroid for which a close encounter with the perturbing body occurs; in this case a conventional ground-based astrometry is used.
2. Very close encounters of some asteroids with space probes; as a result more sophisticated and precise onboard observations allow very accurate and reliable estimates of the masses of these asteroids.
3. Several of the largest asteroids affect the motion of Mars so strongly that their masses can be estimated from an analysis of ranging to the martian landers.

Until now masses of about 100 asteroids have been obtained by the dynamical methods with various levels of accuracy achieved. Masses of the overwhelming majority of the other asteroids are too small to be determined by the dynamic methods in which the ground-based astrometry is used; however, their total impact on the motion of Mars and Earth is not negligible. Fortunately nowadays the astrophysical methods are greatly improved and have provided a data set from which the masses may be estimated for about 2000 asteroids. A very important contribution to this problem was made after launching the dedicated satellite IRAS was launched, which measured the infrared fluxes from a large number of asteroids.

In the work by Krasinsky *et al.* (2001) we have compared the masses obtained by a number of authors based on the method of the close asteroid encounters with the IRAS-based masses. It appears that, excepting the three largest asteroids (Ceres, Pallas, and Vesta) and seven other minor planets, the astrophysical method gives much more accurate estimates (at least by one order). In the cases when the dynamical estimates appear satisfactory they are in good agreement with IRAS data. The successful dynamical determinations based on close encounters of minor planets have been produced only when very favorable conditions have been fulfilled (several encounters, long durations of the astrometric data, or very high accuracy of the data attained by the space telescope Hipparcos; (see Bowell *et al.* 1994, Carpino and Knezevic 1996, Goffin 1991, Krasinsky *et al.* 2001, Kuznetsov 1999, Michalak 2000, 2001, Viateau and Rapaport 1998, Viateau 2000, Viateau and Rapaport 2001).

So the present study is based on the masses derived from IRAS data. The astrophysical method being applied to the three largest minor planets gives wrong results because these planets have complicated internal structures and their mean densities cannot be restored reliably from their spectral classes. Fortunately it appears possible to derive rather accurate masses of these asteroids in the process of fitting the planetary ephemerides to the ranging observations from perturbations upon the orbit of Mars (Standish and Hellings 1989, Standish 2000, Pitjeva 1997, Pitjeva 2001a). Masses obtained in this way are in good agreement with a number of dynamical estimations based on analysis of encounters with other asteroids.

At present 3316 radii, obtained by the astrophysical method, are published in the open NASA database SBN ("Small bodies node of the NASA Planetary Data System" available at http://pdssbn.astro.umd.edu). This set includes both the IRAS data (1991 entries) and results of some ground observations. A taxonomic code from Tholen (1989) is also given which allows one to estimate the densities of 1098 asteroids. These data may be checked and appended by the radii and taxonomic codes for about 300 asteroids from Bowell *et al.* (1979), Tedesco *et al.* (1989), Howell *et al.* (1994), Xu *et al.* (1995), and Barucci *et al.* (1997). We referred Tholen's taxonomic codes to the three compositional taxonomic types making use of the compositional interpretation of the asteroid taxonomy types after Bell *et al.* (1989). These types are carbonaceous (C), sillicate (S), and metallic (M). The adopted correspondence is given in Table I.

[Table I]

When the ephemerides DE403/DE405 were constructed (Standish *et al.* 1995, Standish 1998) mean *a priori* densities of each of the three types were used to calculate the perturbing accelerations from 297 asteroids, selected because they have a relatively large effect upon the motion of Mars. These densities have been revised in the process of fitting these ephemerides to observational data in Standish (2000) (Table I, next line). The densities given in the last two lines of Table I have been derived in a similar way from an independent analysis of practically the same observational data (measurements of distances to the landers and surfaces of the inner planets) while the numerical ephemerides EPM2000 are constructed (Pitjeva 2001a, Pitjeva 2001b) (Table I, line 4) and from the present analysis in which the perturbations from the solid ring are accounted for (Table I, line 5). We use 1σ uncertainty for statistical errors.

The starting system of the masses of the minor planets was that of DE405; the perturbing 297 asteroids, their radii, and taxonomic classes were selected by J.G.Williams (1988, private communication to E. M. Standish who kindly provided us with these data).

At present with the available IRAS radii and Tholen's classes of the asteroids it seems possible to determine the masses for a larger set of the asteroids to compute the perturbations from these bodies more accurately than has been computed in DE403/DE405. So an attempt has been undertaken to account for perturbations from 351 asteroids making use of the published IRAS data. The results are controlled by comparison with the masses adopted in DE405, and by fitting corresponding planetary ephemerides to the lander ranging data and other measurements (see Section III).

A priori estimates of the errors of IRAS-based radii vary in the range from 10% (for large asteroids) to 35% (for small asteroids). It seems plausible that for any individual asteroid of known spectral class the error of densities given in Table I may be as large as 20%. The corresponding total error of the individual mass may reach 30% for large asteroids and 100% for small asteroids.

In the following two cases the *a priori* estimate may be checked by comparison with the masses of the minor planets, (433) Eros and (253) Mathilde, determined by the space mission NEAR to these asteroids (Yeomans *et al.* 1997, Yeomans *et al.* 1999). The NEAR results and those based on IRAS data are presented in Table II.

[Table II]

The errors of NEAR masses may affect only the last decimals of the values given in Table II. One can see that the supposed error of IRAS masses for small asteroids is supported by the comparison with the NEAR results.

The mass distribution derived from the IRAS data making use of our estimates of the densities (the last line of Table I) is presented in Fig. 1. The asteroids are ordered as their masses diminish ($m_1 > m_2 > ... > m_N$) and the consequent sum of the masses $M = \sum_1^N m_i$ in the units $10^{-10} M_{\odot}$ is depicted as a function of N. The arrows on the curve mark the mass of Ceres and the total mass of the asteroids accounted for in DE405. One can see that a small but noticeable part of the asteroids which affect the motion of Mars had not yet been taken into account.

[Figure 1]

III. DYNAMICAL ESTIMATIONS OF THE MASS OF THE ASTEROID BELT

In our analysis of the ranging data the initial values of the astronomical constants involved were taken from DE405. They have been derived as a result of fitting to radiometric observations during 1964–1994 presented on the website of Commission 4 IAU (http:/ssd.jpl.nasa.gov/iau-comm4/) without Pathfinder and Mars Global Surveyor data; the cited above JPL Interoffice Memoranda (Standish *et al.* 1995, Standish 1998) as well as (Pitjeva 2001b) also may be found on this site. This data set was complemented with Russian measurements of ranging to the inner planets (1961–1995) and the Pathfinder lander. For our aims the measurements of ranging to the martian landers Viking-1, Viking-2, and Pathfinder are of a paramount importance.

As was mentioned above the astrophysically derived masses have allowed usto improve significantly the accuracy of the planetary ephemerides. Unfortunately, adding new perturbing asteroids to progress further appears to be not a routine work as our experiments have shown. In fact the SBN database keeps two sets of radii: a set derived from IRAS observations (they are referred hereafter as the "radiometric radii" or " system 1 of radii") and a larger set from the IMPS Ground Data File ("system 2"). In the SBN there is no reference concerning these sets; information about the sets is given in Tedesco (1992). Our analysis has shown that in most cases these two sets are in good agreement and only a slight scaling is needed to transform SBN radii R (in kilometers) to the radiometric radii R_r:

$$R_r = (0.966 \pm 0.001)R - (0.313 \pm 0.027). \quad (1)$$

However, there are two families of minor planets (see Fig. 2) for which the radii strongly disagree for the two sets. In both these cases the linear dependence of R_r on R again takes place but with very different values of the coefficients,

$$R_r = (0.426 \pm 0.013)R - (0.713 \pm 0.446).$$

for the family B, and

$$R_r = (0.244 \pm 0.008)R - (0.330 \pm 0.138)$$

for the family C.

[Figure 2]

In Fig. 2 the straight line A corresponds to relation (1), the lines B and C present the linear dependence for the two other families. The families A, B, and C consist of 1555, 53, and 36 asteroids respectively. It appears that all the asteroids from family B are of the same taxonomic class C, while those from family C belong to either S or M classes. It follows from Tedesco (1992) that some asteroids of system 2 have default albedos, and the lowest albedo 0.01, known for asteroids, has been used for them. As a result radii of these asteroids are overstated (in particular it is true for all asteroids of families B and C). Unfortunately several of the additional asteroids appear to belong to these families and have somewhat uncertain estimates of their masses. In our analysis (Krasinsky *et al.* 2001) it has been shown that the best values of radii of the 300 asteroids (perturbations which are accounted for) correspond namely to system 2. Thus the radii given by system 1 must be corrected making use of relation (1) obtained by comparing the two systems of radii for the asteroids of family A.

So, when the radar measurements are processed several solutions could be obtained:

1. Solution 1: 300 asteroids with the masses estimated independently from the radiometric data set of system 1, corrected for scale. For 16 asteroids which have no radiometric radii in the SBN database, the masses have been taken from the set used for DE405.
2. Solution 2: the same set of asteroids as in Solution 1 with simultaneous estimation of the mass and radius of the asteroid ring.
3. Solution 3: 300 asteroids and the next largest 51 asteroids of the main belt for which radiometric radii are given in the SBN database (system 1), plus

a massive asteroid ring to represent the smallest asteroids.

4. Solution 4: 300 asteroids and another set of 51 asteroids of the main belt with any SBN available radii with an estimation of the doubtful masses of file asteroids, plus the massive asteroid ring.

In producing these solutions the complete set of ranging observations has been processed, corrections to the densities of the three classes (C, S, M) being included in the list of the solve-for parameters. Lines 1–3 of Table III present the rms σ (in meters) of the postfit residuals for the ranging observations of Viking-1, Viking-2, and Pathfinder. In the next six lines the estimated densities ρ_C, ρ_S, ρ_M (in g cm^{-3}) of the three classes of asteroids and the corresponding errors are given. The large error ± 1.35 of ρ_M for Solution 4 is due to strong correlations (up to 90%) with additional parameters of this solution (namely with corrections to the masses of the asteroids Polyhymnia, Angelina, Atala, Ismene, and Ludmila).

[Table III]

In Solution 1 the mass of the ring was set equal to zero, in Solution 2-4 the mass of the ring (in $10^{-10} M_\odot$) and its radius (in AU) were included in the list of estimated parameters. Statistically significant estimates have been obtained for both the mass M_{ring} (with the formal errors about $0.5 \cdot 10^{-10} M_\odot$) and mean radius R_{ring} of the ring (with the formal error 0.06 AU). M_{tot} gives sum of the masses of the asteroids from which perturbations were individually accounted for during integration. From the dispersion of results for the different solutions it seems that the most plausible estimate of the mass M_{sum} of all asteroids is

$$M_{sum} = (18 \pm 2) \cdot 10^{-10} M_\odot . \quad (2)$$

IV. DISTRIBUTION OF ASTEROID MASSES

To check this estimate we can apply a theoretical distribution of the number $N(r)$ of minor planets with radii exceeding r (the distribution is based on a collisional model of fragmentation described in Dohnanyi (1969), Hawkins (1959), *Minor planets* (1973). For the density $dN(r)$ of this distribution the following expression holds true

$$dN(r) = -\beta r^{-3.5} dr, \tag{3}$$

where $\beta > 0$ is a constant.

Let $M(r)$ be the total mass of all asteroids with radii greater than r. Supposing that some mean density ρ may be used to calculate the masses of asteroids from their volumes, we obtain after integration the following expression for the distribution $M(r)$

$$M(r) = \rho \int \frac{4}{3}\pi r^3 dN(r) = \beta_1 r^{0.5} + \beta_0, \tag{4}$$

with some constants β_0 and β_1. The constants β and β_1 are connected by the relation

$$\beta_1 = -\frac{8\pi}{3}\rho\beta. \tag{5}$$

Now we can evaluate the constants β_0, β_1 by fitting the distribution to the set of 300 asteroids studied above whose masses and radii provide the best data for such estimating. We assume that there are no significant systematic observational selection effects in this region of changing of r where the asteroids are large enough. Then we can extrapolate the derived distribution for $r \to 0$ and compare $M(0)$ with estimate (2).

After fitting we have obtained for the value $M(r)$ the following expression

(r is in kilometers, M is in $10^{-10}M_\odot$):

$$M(r) = 17.65 - 0.829\sqrt{r}. \tag{6}$$

We see from this distribution that the total mass $M(0)$ of the asteroid belt is $17.6 \cdot 10^{-10}M_\odot$, whose value is in excellent agreement with our finding based on the study of the perturbations upon the orbit of Mars. In Fig. 3 both the curve of this distribution and the experimental data (for about 2000 minor planets) are depicted.

[Figure 3]

In fact the fragmentation models predict for the power index taken as -3.5 in (3) the values in the range from -3.53 to -3.47. We have constructed the mass distributions given by Eq.(4), varying the power index for the lower and upper boundaries of this interval. The corresponding changes in the estimation of the total mass $M(0)$ appear to be about $\pm 0.4 \cdot 10^{-10}M_\odot$ whose value is within the uncertainty of $M(0)$.

V. HIDDEN ASTEROID MASS

It seems useful to express the right part of distribution (3) in terms of the absolute magnitude H; thereafter it would be possible to compare it with all of the numbered asteroids. We apply the relation between radius r (in kilometers) of an asteroid and H:

$$\log r = 3.1 - 0.2H, \tag{7}$$

taken from Chebotarev and Shor (1976).

Then instead of (3) we obtain

$$dN \approx 0.2 \ln 10 \ \beta r^{-2.5} dH = 0.2 \ln 10 \ \beta 10^{-2.5(3.1-0.2H)} dH. \tag{8}$$

The parameter β in distribution (3) may be calculated now with the help of Eq. (5) in which $\beta_1 = -0.829$ as it follows from (6). Here the mean density $\rho = 1.7$ g cm^{-3} has to be expressed in the units $10^{-10} M_\odot \mathrm{km}^{-3}$. With $M_\odot = 1.99 \cdot 10^{33}$ g we have $\rho = 0.0085 \cdot 10^{-6}$. Thus we obtain

$$\beta = 11.6 \cdot 10^6.$$

With this value for β distribution (3) shows that the expected number of minor planets with radii of 1 km is $12 \cdot 10^6$ and it is about 9000 when the radii are in the range from 10 to 20 km. The typical mass of asteroids from the last subset is about $0.0001 \cdot 10^{-10} M_\odot$ with the total mass $\approx 1 \cdot 10^{-10} M_\odot$ whose value is not negligible if one processes the observations of the landers. For a unit interval of magnitudes, and in the logarithmic scale, expression (8) becomes

$$\log dN \approx 0.5H - 1.02. \tag{9}$$

In Fig. 4 the line marked by the symbol "A" presents this relation in the plane $(\log dN, H)$. The black circles are the values obtained after the total number of minor planets in the intervals of the magnitudes $(H, H+1)$ are calculated. (The experimental data are taken from the data set at our disposal that includes about 30,000 numbered asteroids).

[Figure 4]

One can see that for $H < 8$ the slope of the theoretical line corresponds to the experimental dependence of $\log dN$ on H. Assuming that there is no observational selection for $H < 8$, we can calibrate the dependence given by (9) by the experimental data in this region of magnitudes. Then instead of (9) we obtain

$$\log dN \approx 0.5H - 1.80. \tag{10}$$

The line marked by the symbol "B" corresponds to this functional dependence. It is interesting that the distribution (10) can be obtained after a small constant correction of magnitude 0.3 is applied to the starting dependence of $\log r$ on H (7). Then instead of (7) we have to set

$$\log r = 3.4 - 0.2H.$$

Such a small correction is well within the uncertainty of relation (7) (for instance due to the adopted albedo or the photometric system).

The data presented by Fig. 4 make it possible to estimate the expected number of minor planets in the asteroid belt which have not yet been discovered in given intervals of absolute magnitudes (see Table IV). In the last column of Table IV the ratio of the discovered minor planets in the unit intervals of absolute magnitude to the expected number of planets in this interval is given (in percents). One can see that the expected number of asteroids of the main belt for which the magnitude $H < 14$ is about 130,000, and about 10% of such asteroids have already been discovered.

[Table IV]

VI. CONCLUDING REMARKS

The main conclusions of this study may be summarized in the following way.

There is evidence that in the asteroid belt a hidden mass exists that reveals itself by measurable effects in the motion of Mars. Perturbing effects of a large number of the unobserved small asteroids which contribute to this mass may be effectively described by the model of a perturbing ring. The total mass of all asteroids of the asteroid belt (including all discovered asteroids) is estimated as $(18 \pm 2) \cdot 10^{-10} M_{\odot}$. This estimate is in good agreement with the theoretical distribution of asteroid masses given by fragmentation theory and based on values of parameters of the distribution derived from contemporary values of the masses of the 300 largest asteroids. A significant part of the minor planets of the asteroid belt is too small to be observed from the Earth but their hidden masses produce measurable effects in the motion of Mars.

In recent work (Jedicke and Metcalfe 1998) results of a new global asteroid survey are discussed. It is supposed that the survey is complete to about absolute magnitude $H \approx 12$ and argued that the power index in the distribution $N(r)$ (Eq.(3)) is not constant and differs from the value given by the fragmentation theory of Dohnanyi (1969). If such interpretation of the survey is true then our prediction for the expected number of the minor planets (at least for smaller asteroids) seem to be overestimated. Moreover in this case our value of the summary mass of the asteroid belt (Eq.(2)) derived from the dynamical studies also may be suspected as too large. The present work is probably a first attempt to estimate the hidden mass in the asteroid belt by direct analysis of very small perturbations in the motion of the major planets; in future new precise positional measurements provided by the ongoing NASA program of Mars exploration will make it possible to essentially improve the accuracy and reliability of the dynamical estimate of the hidden mass. Indeed, after this work has been completed new measurements of distances to Mars derived from Mars Global Surveyor satellite became available (see the cited above website of Commission 4). New global solution with the included MGS data yields $M_{ring} = (3.6 \pm 1) \cdot 10^{-10} M_{\odot}$ and $M_{belt} = (16.6 \pm 2) \cdot 10^{-10} M_{\odot}$

for the total mass of the belt. These values only slightly differ from the results presented in this paper.

APPENDIX

PERTURBING FORCE OF THE ASTEROID RING

Let r, v be the radius vector and orbital longitude of a perturbed body, r_p, v_p are those of a perturbing asteroid. The perturbing force $\overline{F}$ acting upon the body is given as

$$\overline{F} = -\mathrm{grad}\ U,$$

where

$$U = gm\frac{1}{\sqrt{r^2 - 2rr_p\cos(v - v_p) + r_p^2}}.$$

If the perturbing asteroids are distributed uniformly along a circular orbit in the plane of the orbit of the perturbed body, then the force function may be presented as the integral

$$U = gM\int_0^{2\pi}\frac{dv_p}{\sqrt{r^2 - 2rr_p\cos(v - v_p) + r_p^2}},$$

where M is sum of the masses of the asteroids. We are interested in the case of the perturbations on the orbits of Mars and the Earth, so $r < r_p$ and the elliptical integral may be expressed in terms of the hypergeometric function F (see Bateman and Erdélyi 1953)

$$U = \frac{gM}{r_p}F\left(0.5, 0.5, 1; \alpha^2\right),$$

where $\alpha = r/r_p$ and the standard notations are used:

$$F(a, b, c; z) = 1 + \sum_{n=1}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}z^n, \qquad (A.1)$$

$$(q)_n = q(q+1)...(q+n-1).$$

For the derivative needed to calculate the gradient the following expression may be applied:

$$\frac{d}{d}F(a,b,c;z) = \frac{ab}{c}F(a+1,b+1,c+1;z).$$

Then for the perturbing force of the ring we obtain

$$\overline{F} = 2\frac{gM\alpha}{r_p^2}\frac{d}{d\alpha^2}F\left(0.5,0.5,1;\alpha^2\right)\mathrm{grad}\ r,$$

or

$$\overline{F} = \frac{1}{2}\frac{gM}{r_p^3}\overline{r}F\left(1.5,1.5,2;\alpha^2\right).$$

The simplest way for calculating the hyperheometric function in the right part of this expression is the straightforward application of relation (A.1). In our case $\alpha^2 \approx 0.2$ and the series quickly converges.

ACKNOWLEDGMENTS

We thank E.M. Standish and J.G. Williams (JPL) for granting necessary information and helpful comments, and V.A. Shor (IAA) for useful discussion of the results.

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Table I. Correspondence of Tholen's classes with densities (in g cm^{-3})

Tholen's classes	C, D, P, T, B, G, F	S, K, Q, V, R, A, E	M
Composition type	C	S	M
A priory density	1.8	2.4	5.0
Revised density, Standish 2000	1.29±0.06	2.71±0.04	5.29±0.53
Revised density, Pitjeva 2001a	1.36±0.03	2.67±0.02	–
Revised density, this work	1.38±0.02	2.71±0.02	5.32±0.07

Table II. Masses of Eros and Mathilde in $10^{-10} M_{\odot}$

	(433) Eros	(253) Mathilde
NEAR	0.00003362	0.00051938
IRAS	0.000058	0.000698

Table III. Impact of minor planets on planetary ephemerides, (σ_i in m, ρ_i in g·cm^{-3}, R in AU, M_i in $10^{-10} M_\odot$).

N	1	2	3	4
σ of Viking-1	7.9	7.8	8.2	7.5
σ of Viking-1	5.6	5.4	5.5	5.7
σ of Pathfinder	3.1	3.0	8.5	3.6
ρ_C	1.38	1.34	1.12	1.74
	±0.03	±0.04	±0.04	±0.11
ρ_S	2.71	2.71	2.82	2.64
	±0.02	±0.02	±0.03	±0.08
ρ_M	5.32	5.30	5.35	4.21
	±0.07	±0.13	±0.30	±1.35
R_{ring}		2.94	2.83	2.77
		±0.06	±0.06	±0.06
M_{ring}		5.3	4.0	4.8
		± 0.5	±0.5	±0.5
N	300	300	351	351
M_{tot}	11.8	11.8	12.0	14.9
M_{sum}		17.1	16.0	19.7

Table IV. Expected (N_p) and observed (N_o) numbers of asteroids.

H	N_o	N_p	%
5-6	10	8	100
6-7	25	28	100
7-8	101	89	100
8-9	210	280	70
9-10	363	900	40
10-11	583	3000	20
11-12	1515	9000	16
12-13	4109	28000	14
13-14	8014	90000	9
14-15	8290	280000	3

The figure captions:

Fig. 1. Summing masses derived from the IRAS data.

Fig. 2. The relation between radii: Rr are the radiometric radii of the system 1 and R are the radii of the system 2 (in km).

Fig. 3. Distribution of masses $M(r)$ versus their radii r, $M(r)$ is the total mass of all asteroids whose radius greater than r (in km).

Fig. 4. Distribution of log(N) versus magnitudes H.

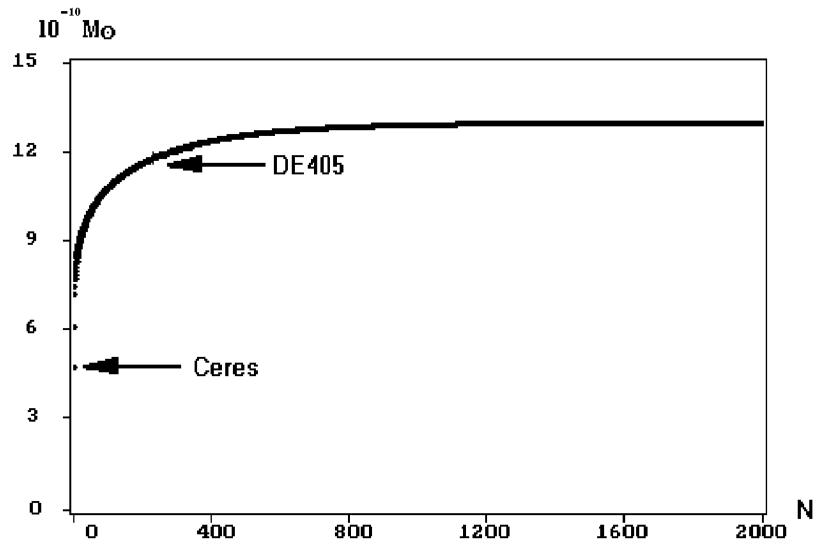

FIG. 1. Summing masses derived from the IRAS data.

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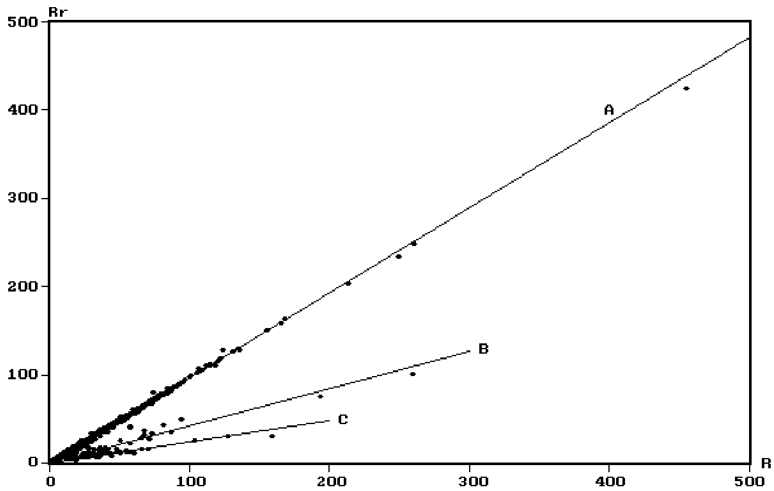

FIG. 2. The relation between radii: Rr are the radiometric radii of the system 1 and R are the radii of the system 2 (in km).

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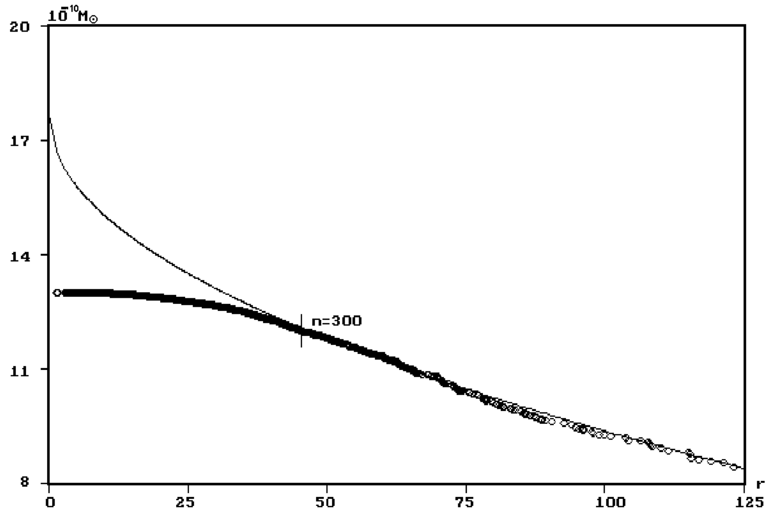

FIG. 3. Distribution of masses $M(r)$ versus their radii r,

$M(r)$ is the total mass of all asteroids whose radius greater than r (in km).

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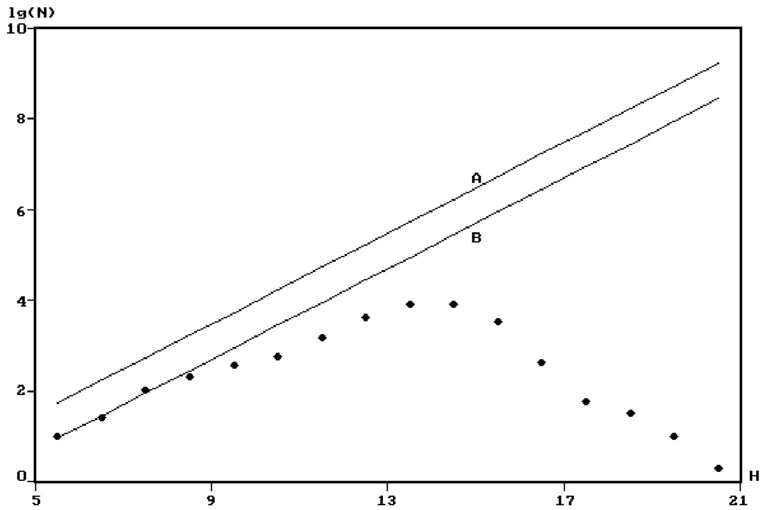

FIG. 4. Distribution of lg N versus magnitudes H.

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